Amplifying Left-Handed Transmission Line Media

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Introduction

Negative-index materials (NIMs), also known as left-handed (LH) media, first postulated by Veselago [1], are becoming an exciting reality, particularly as they are being demonstrated in microwave circuits [2]. The range of applications for NIMs is extensive, and opportunities abound for development of new and powerful imaging techniques throughout the electromagnetic spectrum. The primary drawback of current NIMs is their considerable loss, which renders the results ambiguous and the materials all but useless for practical applications.

Based on our previous work [3-5], we propose a new means of addressing these losses in a scalable format: parametric amplification in LH nonlinear transmission line (NLTL) media. Thus, we introduce a new amplifying NIM where energy in a pump wave at one frequency is transferred to energy in a weak signal wave at another frequency and so amplifies it. We wish to take advantage of the contradirectional parametric interactions, also known as backward wave parametric interactions proposed in optics [6], which were claimed to provide efficient generation and amplification. Though parametric generation in LH NLTLs has been discussed recently in [4] and [7], the possibility of parametric amplification has not yet been demonstrated, and no data on the level of amplification that can be achieved using them has been reported until now.

Here we provide simulations of parametric amplification in a 1D LH NLTL, each segment of consisting of a reverse biased diode and an inductor. Such structures can be fabricated by replacing the capacitors in the LH transmission line medium discussed in [8] and [9] with varactor diodes, as it was suggested in [3] and [4] and realized in [5].

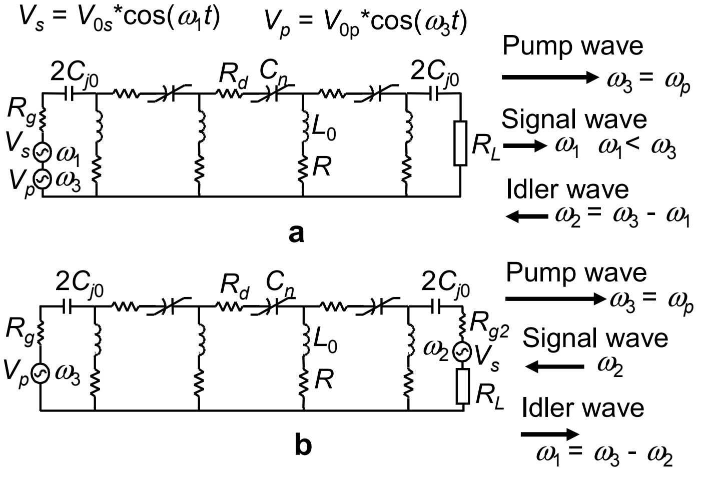

Fig. 1. Schematic circuit diagrams of the LH NLTL systems operating as parametric amplifiers (only three sections are shown).

a) Configuration where signal wave co-propagates with pump wave.
b) Configuration where signal wave is backward towards the pump wave.

Model used in simulations

Equivalent circuit diagrams of the LH NLTLs used in our simulations are shown in Fig. 1. Nonlinear capacitors $C_n = C(V_n - V_{n-1})$ are formed by reverse biased varactor diodes, to provide capacitance-voltage characteristics expressed as:

$$C(V) = \frac{C_{j0}}{\left(1 + \left|(V + V_R)/V_{j0}\right|\right)^M} \qquad (1)$$

In our simulations, the values for prototype Agilent TC803 hyperabrupt varactor diodes are used: $C_{j0} = 1$ pF, $M = 1.0$, $V_{j0} = 0.7$ V, $R_d = 6$ Ω. The reverse bias voltage V_R is chosen to be 4 V. The circuit is loaded with resistance $R_L = Z_0$ ($Z_0 = (L_0/C_{j0})^{1/2}$), which is equal to the generator resistance $R_g = R_L$. A simple 7-diode LH NLTL (only 3 diodes are shown in Fig. 1 for simplicity) has been simulated using the commercial microwave circuit simulator Agilent ADS™.

Parametric interaction in left-handed media

It has been predicted that distributed parametric amplifier or oscillator circuits could have superior stability of operation and efficiency over lumped parametric circuits [10]. However, it is also known that the parametric generation and amplification in dispersionless RH NLTLs is suppressed by shock wave formation [11], [12]. The impossibility of shock wave formation in LH NLTLs enables a variety of parametric processes. Effective parametric interaction in medium exhibiting a second-order nonlinearity generally requires phase-matching of three waves. Our simulations have shown that the anomalous dispersion of LH NLTL system enables effective parametric interactions of the type:

$$\omega_1 + \omega_2 = \omega_3, \ \beta_1 - \beta_2 = \beta_3$$

when in the "parametric oscillator configuration", the high-frequency pump wave with frequency ω_3 and wavenumber β_3 is excited by the voltage source connected at the input port of LH NLTL and it generates two other waves with frequencies ω_1 and ω_2 such that $\omega_1 < \omega_2$ and $\omega_1 + \omega_2 = \omega_3$. The wave with frequency ω_2 propagates in the opposite direction relative to the pump wave and the wave having frequency ω_1. We therefore have a similar situation to backward wave parametric generation [7], [6]. The backward-propagating parametrically generated wave ω_2 enables internal feedback resulting in a considerable energy transfer from the pump wave to the parametrically excited waves.

Parametric amplification in LH NLTL

If the amplitude of a high-frequency pump wave exceeds a certain threshold value it may parametrically generate two other waves. This threshold value depends on the loss in the LH NLTL, its length and the boundary conditions (matching) at the input and output. No parametric generation occurs when the amplitude of the voltage source is below this value. However, when a weak signal wave is fed into the LH NLTL together with the pump wave, a parametric amplification is observed. In this case, we have two input waves: an intense pump wave and a weak signal wave. The power from the pump wave is transferred to the signal wave, and so amplifies it. A third parasitic idler wave is generated to provide phase matching. To use the LH NLTL as a parametric amplifier, two sources are connected as inputs to the system, in order to generate a high power pump wave and a weak signal to be amplified. In fact, two configurations are possible. Fig. 1*a* illustrates an example of a parametric amplifier configuration where a weak signal wave

($\omega_s = \omega_1$) propagates in the same direction as the high-frequency pump wave and, hence, both the pump wave and the signal wave to be amplified should be generated at the input of the LH NLTL. To achieve this, the pump voltage source $V_p = V_{0p}\cos(\omega_p t)$ and the signal voltage source $V_s = V_{0s}\cos(\omega_s t)$ are connected together in series and coupled to the input terminals of the LH NLTL. In the second configuration shown in Fig. 1*b*, the signal wave ($\omega_s = \omega_2$) is backward towards the pump wave having frequency ω_3, and these waves are excited at the opposite sides of the LH NLTL.

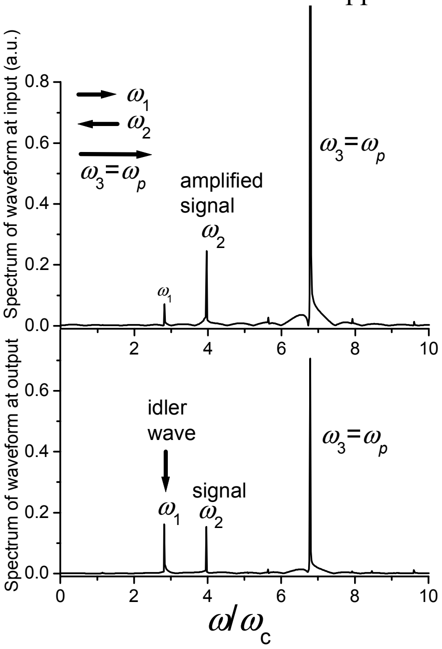

Fig. 2. Parametric amplification in the backward wave amplifier configuration shown in Fig. 1*b*: spectrum of the waveforms at the loads $R_L/Z_0 = 0.95$ connected at the input and at the output of 7-section LH NLTL ($V_{0p} = 2.9$ V, $\omega_p/\omega_B = 6.78$, $V_{0s} = 0.3$ V, $\omega_s/\omega_B = 3.94$, $\omega_B = 1/(2(L_0 C_{j0})^{1/2})$).

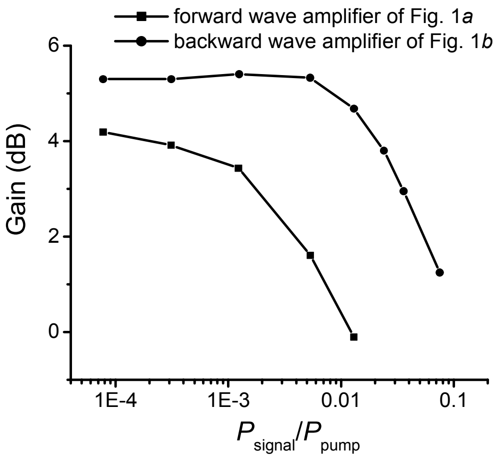

Fig. 3. Gain as a function of the power of the signal wave normalized to the power of the pump wave for configurations shown in Fig. 1*a* and 1*b*. ($\omega_s/\omega_B = 2.865$ for configuration shown in Fig. 1*a* and $\omega_s/\omega_B = 3.94$ for configuration shown in Fig. 1*b*, $V_{0p} = 2.9$ V).

The simulations were carried out for both parametric amplifier configurations and demonstrate that the second configuration (Fig. 1*b*) exhibits superior performance over the first configuration (Fig. 1*a*). Fig. 2 illustrates amplification of a weak signal in the second configuration. It shows the spectra of the voltage waveforms at the input and at the output for the amplitude of the pump wave of $V_{0p} = 2.9$ V, which is below the parametric generation threshold. Fig. 3 shows the gain as a function of the power of the signal wave normalized to the power of the pump wave for the configurations shown in Fig. 1*a* and *b* for the same value of the amplitude of the pump wave ($V_{0p} = 2.9$ V). Both configurations provide gain within some range of the input power of signal wave. In other words, there exists a restriction on the power of the signal that can be amplified. Gain drops rapidly when the signal power exceeds a certain value and the system starts operating as a parametric oscillator. The second configuration provides higher amplification, "flatter" dependence of gain on signal power and has a wider operating range over which the circuit provides linear gain. Thus, the second configuration is more attractive. The simulations predict a 5.3 dB/7 section amplification of the backward signal wave. It should be mentioned that if the same signal wave is fed into the LH NLTL without an intensive pump wave present, this signal would be attenuated by 3.5 dB. Thus, the interaction of the weak signal with the intensive pump wave in the LH NLTL will suppress the 3.5 dB attenuation caused by the diode spreading resistance and will instead provide 5.3 dB gain.

The anomalous dispersion of the LH medium allows phase matching of the parametrically interacting waves so that the “coherence length” of the nonlinear parametric processes described above is large and thus the amplitude of the parametrically amplified waves should grow with distance. We therefore expect higher amplification to occur in longer LH NLTLs.

Conclusions

Using a one-dimensional structure as the basis for our studies, we have demonstrated that the LH NLTL that we developed for slightly different applications can also, under the correct design conditions, be used for parametric amplification, by which energy in a pump wave at one frequency is transferred to energy in a weak signal wave at another frequency. Our approach could be scaled from its current X-band (10 GHz) form into THz, infrared, or ultimately visible form. Moreover, extending our results to a 2-D structure and taking advantage of the orthogonality between pump and signal waves that is possible in a two-dimensional system would enable amplification embedded within the NIM, overcoming the losses that dominate their performance today. This will open up a host of possibilities for applications, including sub-wavelength stand-off imaging, lenses that brighten the image, and conformal radar antennas that both break the diffraction limit and contain integrated low-noise amplifiers for direct processing of the image.

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