Higher Harmonic Generation and Parametric Instabilities in Left-Handed Nonlinear Transmission Lines

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Introduction

Negative-index materials (NIMs), also known as left-handed (LH) media, first postulated by Veselago [1], are becoming an exciting reality, particularly as they are being demonstrated in resonant RF and microwave circuits [2-5]. The majority of studies and applications of LH media to date are in the linear regime of wave propagation. However, materials that combine nonlinearity with the anomalous dispersion exhibited by LH media, can give rise to many new and interesting phenomena and applications [6-10]. In [11] we presented a theoretical investigation of the basic nonlinear wave propagation phenomena in LH medium, which is based on the dual of the conventional nonlinear transmission line (NLTL), a left-handed (LH) NLTL with anomalous dispersion. Here we present experimental results for harmonic generation and parametric generation in a short left-handed NLTL that confirms the general predictions of [11].

Qualitative comparison of nonlinear waveform evolution in RH and LH NLTL

Nonlinear waveform evolution in LH NLTLs is qualitatively different from waveform evolution in conventional right-handed (RH) NLTLs. Propagation of a sinusoid wave in RH NLTLs results in edge steeping due to formation of a shock wave formation so that the spectrum of the waveform at the TL output contains the fundamental frequency together with its higher harmonics, whose amplitudes decay with frequency [12]. Harmonic generation in a LH NLTL is possible in a higher frequency range than in the dual RH NLTL with all other parameters being the same because of the high-pass nature of the structure. The amplitude of the second harmonic in the *n*-th section of a periodically loaded transmission line is [11]

$$V_{2,n} \sim \sin\left((\beta_2 - 2\beta_1)n\right) \tag{1}$$

In the case of a RH NLTL, the value of $|\beta_2 - 2\beta_1|$ is small and hence the amplitude of the second harmonic grows linearly with distance; the optimal length of the NLTL is determined by the tradeoff between this linear growth and the exponential decay due to loss along the line. In the case of a LH NLTL, $|\beta_2 - 2\beta_1|$ is large due to anomalous dispersion. This gives rise to a highly localized energy exchange between the fundamental wave and its second harmonic. It is apparent from (1) that maximum amplitude of the second harmonic is achieved when $(\beta_2 - 2\beta_1)n = (2k+1)\pi$ at the end of the line, therefore, the optimal number of LH NLTL sections is

$$N_{opt} = \frac{(2k+1)\pi}{\beta_2 - 2\beta_1}, \quad k = 0,1,2,3.. \tag{2}$$

Theoretical analysis of the second harmonic generation in LH NLTL shows that, despite the large phase mismatch in LH NLTLs, the conversion efficiency can be higher in the case of LH NLTLs versus RH NLTLs for short NLTLs.

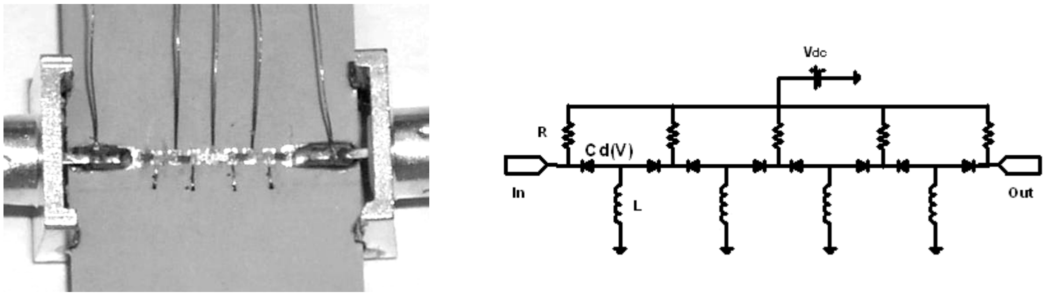
Though harmonic generation dominates in short LH NLTLs, the impossibility of shock wave formation in LH NLTLs gives rise to a variety of parametric processes in longer LH NLTLs that compete with harmonic generation. The anomalous dispersion of LH NLTLs enables effective basic parametric interactions of the type:

$$\omega_1 + \omega_2 = \omega_3, \ \beta_1 - \beta_2 = \beta_3$$

when the high-frequency pump wave with frequency ω_3 and wavenumber β_3 generates two other waves with frequencies ω_1 and ω_2 such that $\omega_1 < \omega_2$ and $\omega_1 + \omega_2 = \omega_3$. The wave with frequency ω_2 propagates in the opposite direction relative to the pump wave and the wave having frequency ω_1. Thus, the waveform evolution in LH NLTL is generally a result of competition between harmonic generation and parametric generation.

LH NLTL design

In order to demonstrate basic nonlinear wave propagation phenomena we fabricated a 4-section LH NLTL shown in Fig. 1. It was realized on Rogers RT/Druid 3010 board (ε_r = 10.2). MACOM hyper-abrupt junction GaAs flip-chip varactor diodes (MA46H120) were attached using conductive silver epoxy. The C_{j0} of the diode, as described on manufacturer's data sheet is 1.1 pF with capacitance ratio $C(0V)/C(10V) = 7.5$. Inductances were implemented by connecting 0.12 mm diameter copper wires to the ground plane on the back side of the board. The length of this wire was chosen to provide inductance of 1.25 nH. DC bias wires are connected with 3 kΩ resistors between diodes as shown in Fig. 1 (b). The total circuit size was 10 x 2 mm, neglecting the connectors and bias wires. Measurement indicated –6 dB cut-off frequencies at 2.7 GHz and 4.1 GHz for 0 V-bias and 4 V-bias, respectively.



(a) (b)

Fig. 1: (a) Fabricated LH NLTL and (b) equivalent circuit.

Nonlinear wave propagation phenomena in LH NLTL

Fig. 2 shows the spectrum measured with an Agilent E4448A Spectrum Analyzer from the output of 4-section LH NLTL (6 dB attenuator was in line for protection) for different values of bias voltage. The typical spectra of the output waveform corresponding to the maximum of the second harmonic are shown in Fig. 2 (a) and (d). A fundamental of 2.875 GHz generates numerous higher harmonics, and the second harmonic dominates over the fundamental and the other harmonics due to intensive Bragg reflection of the fundamental wave. Thus, the LH NLTL combines the properties of both a harmonic generator and a bandpass filter, and under certain conditions may provide an almost pure higher harmonic at its output. The best result for second harmonic conversion efficiency measured in this 4-section LH NLTL was 19 % at 2.875 GHz, 17.9 dBm and reverse bias voltage 6.41 V. The second harmonic power delivered into a 50 W load was 11.8 dBm. The conversion efficiency is comparable with the per-stage efficiency of a hybrid hyperabrup Schottky-diode RH NLTL operated in a lower frequency range (as reported in [13]).

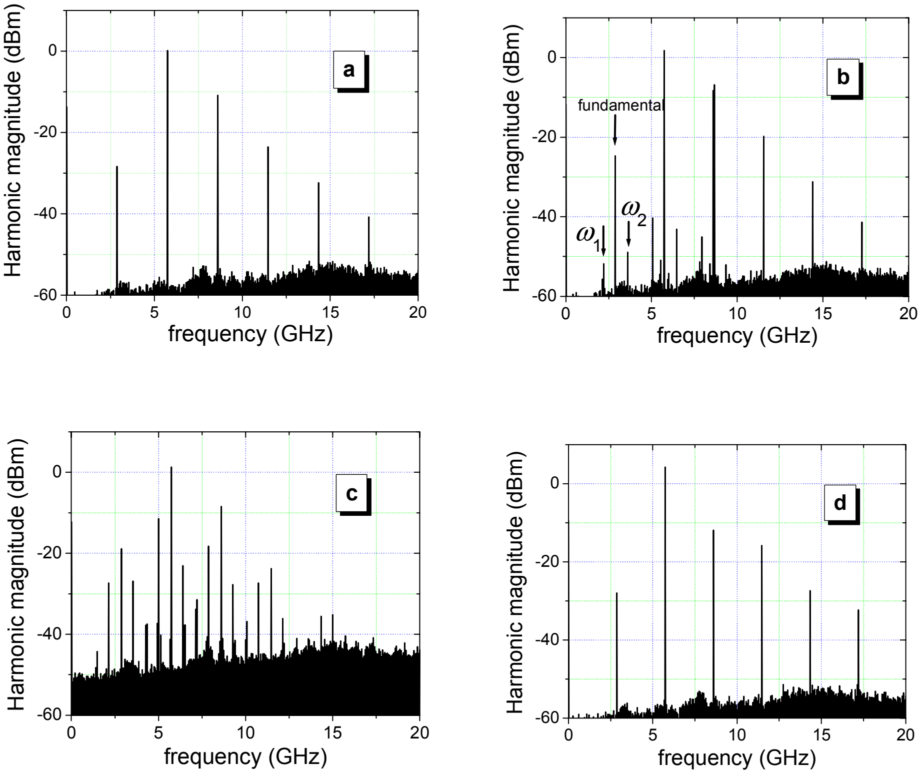


Fig. 2. Spectra of the output waveform generated by a 4-section LH NLTL fed by 2.875 GHz, 19 dBm input signal for different values of the bias voltage (*a* – 4 V, *b* – 4.95 V, *c* – 5 V, *d* – 6.3 V).

Harmonic generation competes with different parametric processes that can make harmonic generation unstable. A wave at 2.875 GHz cannot parametrically generate any other waves since they would exist below the line's cutoff frequency. However, an intensive second harmonic wave may initiate parametric process. The second harmonic at 5.75 GHz excites waves with frequencies 2.2 GHz and 3.6 GHz depicted in Fig. 2 (b) as ω_1 and ω_2. This basic parametric process then initiates multiple higher-order parametric interactions resulting in multiple peaks in the spectrum of the output waveform. The progression of this process is shown in Fig. 2 (c), which illustrates conversion of a monochromatic input signal into a wideband output. Further increase of the reverse bias voltage leads to the stabilization of the harmonic generation and suppression of parametric instability (Fig. 2 (d)).

Conclusions

Our measurements demonstrate efficient higher-harmonic generation along LH NLTLs. Harmonic generation is possible over a significantly wider operating frequency range and at relatively higher frequencies in comparison with the dual conventional lowpass filter NLTLs. Furthermore, LH NLTLs are predicted to have advantages from the design perspective, since we have more freedom to optimize parameters, being much less restricted by the host waveguide structure than in the case of RH periodically loaded NLTLs. They are more compact since the length of the section in practice is determined by the diode. Furthermore, at some parameters (when $|\beta_1| > \omega_1/c$ and $|\beta_3| < 3\,\omega_1/c$ and where ω_1 is the frequency of the fundamental input signal and c is the velocity of light in free space) LH NLTLs can be a waveguide for the fundamental input signal, and acts as a leaky-wave antenna [14, 15] for the generated second harmonic, thus significantly simplifying the radiation of generated power. Extending these results for one-dimensional LH NLTL to higher dimensions would enable combining harmonic generation in LH NLTL media with focusing, due to the negative refractive index of 2- or 3-D LH transmission line media. This may lead to the development of highly efficient and powerful frequency multipliers. The parametric generation and amplification that generally accompany harmonic generation in LH NLTLs will be of interest for building "active" or "amplifying" super lenses based on LH nonlinear medium and provide a means to compensate for the inherent LH medium loss which is a current challenge for existing LH materials.

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